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Asymptotic Synchronization of Leader-Follower Networks of Uncertain Euler-Lagrange Systems

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***Abstract*—This paper investigates the synchronization of a network of Euler-Lagrange systems with leader tracking. The Euler-Lagrange systems are heterogeneous and uncertain and contain bounded, exogenous disturbances. Network communication is governed by an undirected topology. The network leader has a time-varying trajectory which is known to only a subset of the follower agents. A Robust Integral Sign of the Error (RISE) based decentralized control law is developed to guarantee semi-global asymptotic agent synchronization and leader tracking.**

I. INTRODUCTION

Synchronization of multi-agent systems involves matching the state of an agent with that of its neighboring agents in a network and has motivating applications in the fields of physics, engineering, social studies, etc. [1]–[4]. As opposed to typical consensus approaches, synchronization emphasizes similarity of neighbors' states as consensus is being achieved; the difference between a system's state and that of its neighbors is generally included as an error in the control law (c.f. [5]–[7]).

Consensus of networked systems without a leader synchronizes the systems' states but does not allow for control of the consensus value, such as in [8] and [9]. In such approaches, the consensus value is some (often unknown) function of the initial states of the networked systems. Alternatively, synchronization may be performed with a network leader or a specified desired trajectory (often referred to as a virtual network leader), in which all follower systems both synchronize and track a time-varying state. Simultaneous reference tracking and synchronization of a network of nonidentical Euler-Lagrange systems wherein all systems know the desired trajectory are explored in [10]–[12]. Practical applications often contain network topologies in which not every node receives information from the leader; e.g., only vehicles in the front of a formation tracking a vehicle target may be able to observe the location of the target. Synchronization with a leader state which is available to only a subset of the total nodes is examined in results such as [5], [6], and [13]–[15].

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Typical agent synchronization results have focused on single or double-integrator dynamics (c.f. [6], [16]). Motivation to model the systems by nonlinear Euler-Lagrange dynamics arises from the fact that many physical systems can be represented in Euler-Lagrange form [17]. For example, it may be desirable to synchronize the position of multiple robotic manipulators in an assembly line, maintain matching generator phase angles in an electrical grid, synchronize the attitude of distributed satellites, etc. For such applications, results are strengthened by considering heterogeneous Euler-Lagrange systems; i.e., networked robotic manipulators that may not have the same kinematics or dynamics.

Even if the dynamics are structurally the same, unknown exogenous disturbances are inherent to every system and may cause undesirable network performance. Exogenous disturbances are included in the neural network-based adaptive synchronization result in [5] and in the sliding mode-based synchronization result in [6]. The continuous controller in [5] yields a uniformly ultimately bounded result, whereas [6] achieves exponential synchronization through the use of a discontinuous controller.

This paper investigates the synchronization of networked systems consisting of a leader and followers in an undirected topology, where at least one follower is connected to the leader. The networked systems are modeled with Euler-Lagrange dynamics which are nonlinear, heterogeneous, and uncertain. In comparison to results such as [5] and [6], the developed continuous Robust Integral Sign of the Error (RISE) based controller yields semi-global asymptotic decentralized synchronization of the states of the followers to the time-varying state of the leader, despite the effects of bounded exogenous disturbances and model uncertainties. Simulation results are provided for the synchronization of a network of robotic manipulators to illustrate the performance of the developed approach.

II. PROBLEM FORMULATION

A. Preliminaries

Graph theory provides convenient tools to describe the information exchange between multiple agents in a network. Consider a network consisting of one leader and N followers, where the leader is indexed by 0. Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a fixed undirected graph with a non-empty finite set of nodes $\mathcal{V} = \{0, 1, \ldots, N\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. An undirected edge $(i, j) \in \mathcal{E}$ exists if nodes i and j share information. The set of neighbors which have information available to node i is defined as $\mathcal{N}_i \triangleq \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}$. An adjacency matrix

$\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined such that $a_{ij} = a_{ji} > 0$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = a_{ji} = 0$ otherwise. It is assumed that the graph is simple, i.e., $(i,i) \notin \mathcal{E}$, and thus $a_{ii} = 0 \, \forall i \in \mathcal{V}$. Let $\mathcal{D} \triangleq \text{diag}\{D_0, D_1, \ldots, D_N\} \in \mathbb{R}^{(N+1)\times(N+1)}$ be a diagonal matrix, where $D_i \triangleq \sum_{j\in\mathcal{N}_i} a_{ij}$. The Laplacian matrix is then defined as $\mathcal{L} \triangleq \mathcal{D} - \mathcal{A}$.

B. Dynamic Models and Properties

The considered network has $N+1$ agents which have dynamics described by nonidentical Euler-Lagrange equations of motion such that

$$M_0(q_0)\ddot{q}_0 + C_0(q_0,\dot{q}_0)\dot{q}_0 + F_0(\dot{q}_0) + G_0(q_0) = \tau_0 \quad (1)$$

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + F_i(\dot{q}_i) + G_i(q_i) + d_i(t) = \tau_i, \quad i = 1,2,\ldots,N, \quad (2)$$

where the zero index denotes the leader and all other agents, $i = 1,2,\ldots N$, are followers. The terms in (1) and (2) are defined such that $q_j \in \mathbb{R}^m$ $(j = 0,1,\ldots,N)$ is the generalized configuration coordinate, $M_j : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is the inertia matrix, $C_j : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is the Coriolis/centrifugal matrix, $F_j : \mathbb{R}^m \to \mathbb{R}^m$ is the friction term, $G_j : \mathbb{R}^m \to \mathbb{R}^m$ is the vector of gravitational torques, $\tau_j \in \mathbb{R}^m$ is the vector of control input torques to be designed, $d_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ $(i = 1,2,\ldots,N)$ is a time-varying nonlinear exogenous disturbance, and $t \in \mathbb{R}_{\geq 0}$ is the elapsed time.

The following system properties are used in the subsequent analysis.

Property 1. The inertia matrix M_j is symmetric, positive definite, and satisfies $\underline{m}_j \|\xi\|^2 \leq \xi^T M_j \xi \leq \overline{m}_j \|\xi\|^2 \; \forall \xi \in \mathbb{R}^m$ $(j = 0,1,\ldots,N)$, where $\underline{m}_j \in \mathbb{R}$ is a positive known constant and $\overline{m}_j \in \mathbb{R}$ is a known positive function [18].

Property 2. The functions M_j, C_j, F_j and G_j $(j = 0,1,\ldots,N)$ are second order differentiable such that their second time derivatives are bounded if $q^{(k)} \in \mathcal{L}_\infty$, $k = 0,1,\ldots,3$ [19].

The following assumptions are also required for the subsequent analysis.

Assumption 1. The nonlinear disturbance term, d_i, and its first two time derivatives are bounded by known constants for $i = 1,2,\ldots,N$.

Assumption 2. The leader control input, τ_0, is bounded.

Assumption 3. The graph $\mathcal{G}$ is connected.

The leader may be represented by a desired trajectory instead of an actual agent. In this case, the desired trajectory is assumed to be designed such that $q_d^{(k)} \in \mathbb{R}^m$ $(k = 0,1,\ldots,4)$ exists and is bounded [18].

The equation of motion for the follower agents may be written as

$$M\ddot{Q} + C\dot{Q} + F + G + d = \tau, \quad (3)$$

where $Q \triangleq \left[q_1^T, q_2^T, \ldots, q_N^T\right]^T \in \mathbb{R}^{Nm}$, $M \triangleq \text{diag}\{M_1, M_2, \ldots, M_N\} \in \mathbb{R}^{Nm\times Nm}$, $C \triangleq \text{diag}\{C_1, C_2, \ldots, C_N\} \in \mathbb{R}^{Nm\times Nm}$, $F \triangleq \left[F_1^T, F_2^T, \ldots, F_N^T\right]^T \in \mathbb{R}^{Nm}$, $G \triangleq \left[G_1^T, G_2^T, \ldots, G_N^T\right]^T \in \mathbb{R}^{Nm}$, $d \triangleq \left[d_1^T, d_2^T, \ldots, d_N^T\right]^T \in \mathbb{R}^{Nm}$, and $\tau \triangleq \left[\tau_1^T, \tau_2^T, \ldots, \tau_N^T\right]^T \in \mathbb{R}^{Nm}$. For convenience in the subsequent analysis, the leader dynamics are represented as

$$M_\varnothing \ddot{Q}_0 + C_\varnothing \dot{Q}_0 + F_\varnothing + G_\varnothing = \tau_\varnothing, \quad (4)$$

where $Q_0 \triangleq \underline{1}_N \otimes q_0 \in \mathbb{R}^{Nm}$, $M_\varnothing \triangleq I_N \otimes M_0 \in \mathbb{R}^{Nm\times Nm}$, $C_\varnothing \triangleq I_N \otimes C_0 \in \mathbb{R}^{Nm\times Nm}$, $F_\varnothing \triangleq \underline{1}_N \otimes F_0 \in \mathbb{R}^{Nm}$, $G_\varnothing \triangleq \underline{1}_N \otimes G_0 \in \mathbb{R}^{Nm}$, $\tau_\varnothing \triangleq \underline{1}_N \otimes \tau_0 \in \mathbb{R}^{Nm}$, $\underline{1}_N$ denotes an N-dimensional column vector of ones, and $\otimes$ denotes the Kronecker product.

C. Network Properties

The communication topology of the $N+1$ agents is defined by the previously described graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $\mathcal{V} = \{0,1,\ldots,N\}$. The analysis of the follower nodes is facilitated by creating a subgraph $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$, which is formed by removing the leader node and corresponding edges from the graph $\mathcal{G}$. The neighbor set of the follower nodes is defined for all $i \in \bar{\mathcal{V}}$ as $\bar{\mathcal{N}}_i \triangleq \left\{j \in \bar{\mathcal{V}} \mid (i,j) \in \bar{\mathcal{E}}\right\}$.

The adjacency matrix, $\bar{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{N\times N}$, corresponding to $\bar{\mathcal{G}}$ is defined as $a_{ij} = a_{ji} > 0$ if $(i,j) \in \bar{\mathcal{E}}$ and $a_{ij} = a_{ji} = 0$ otherwise with $a_{ii} = 0 \, \forall i \in \bar{\mathcal{V}}$. Let $\bar{\mathcal{D}} \triangleq \text{diag}\{D_1, D_2, \ldots, D_N\} \in \mathbb{R}^{N\times N}$, where $D_i \triangleq \sum_{j=1}^{N} a_{ij}$. The leader-removed graph Laplacian matrix, $\bar{\mathcal{L}} \in \mathbb{R}^{N\times N}$, of $\bar{\mathcal{G}}$ is then defined as

$$\bar{\mathcal{L}} \triangleq \bar{\mathcal{D}} - \bar{\mathcal{A}}. \quad (5)$$

The leader-follower connectivity matrix B completes the communication topology description and is defined as $B \triangleq \text{diag}\{b_1, b_2, \ldots, b_N\}$ with $b_i > 0$ $(i = 1,2,\ldots,N)$ if $0 \in \mathcal{N}_i$ and $b_i = 0$ otherwise. Note that because the graph $\bar{\mathcal{G}}$ is undirected and connected and at least one follower agent is connected to the leader by Assumption 3, the matrix $\bar{\mathcal{L}} + B$ is positive definite and symmetric [20]. The customarily used Laplacian matrix is positive semi-definite for a connected undirected graph; however, the matrix $\bar{\mathcal{L}}$, also known as the "Dirichlet" or "Grounded" Laplacian matrix, is designed such that $\bar{\mathcal{L}} + B$ is positive definite given Assumption 3 [20].

III. Control Objective

The objective is to design a continuous controller which ensures that N agents asymptotically track the state of the reference node with only self and neighbor signal feedback despite model uncertainties and bounded exogenous system disturbances. In addition, the second derivative of the generalized configuration coordinate, i.e., acceleration, is assumed to be unavailable. To quantify this objective, the local neighborhood position tracking error, $e_{1,i} \in \mathbb{R}^m$, is defined as [6]

$$e_{1,i} \triangleq \sum_{j\in\bar{\mathcal{N}}_i} a_{ij}(q_j - q_i) + b_i(q_0 - q_i). \quad (6)$$

The error signal in (6) includes the summation $\sum_{j\in\bar{\mathcal{N}}_i} a_{ij}\left(q_j - q_i\right)$ to penalize state dissimilarity between neighbors and the proportional term $b_i\left(q_0 - q_i\right)$ to penalize state dissimilarity between a follower agent and the leader, if that connection exists. The ability to emphasize either follower synchronization or leader tracking is rendered by assigning $a_{ij} = k_a$ if $a_{ij} > 0$ and $b_i = k_b$ if $b_i > 0$, where $k_a, k_b \in \mathbb{R}$ are constant positive gains.

An auxiliary tracking error, denoted by $e_{2,i} \in \mathbb{R}^m$, is defined as

$$e_{2,i} \triangleq \dot{e}_{1,i} + \alpha_{1,i} e_{1,i}, \tag{7}$$

where $\alpha_{1,i} \in \mathbb{R}$ denotes a constant positive gain. For brevity, let $L_B \triangleq \left(\left(\bar{\mathcal{L}} + B\right) \otimes I_m\right) \in \mathbb{R}^{Nm\times Nm}$. Note that because $\bar{\mathcal{L}} + B$ is positive definite and symmetric, L_B is positive definite and symmetric. The error systems in (6) and (7) may be represented as [5]

$$E_1 = L_B\left(Q_0 - Q\right), \tag{8}$$

$$E_2 = \dot{E}_1 + \Lambda_1 E_1, \tag{9}$$

where $E_1 \triangleq \left(e_{1,1}^T, e_{1,2}^T, \ldots, e_{1,N}^T\right)^T \in \mathbb{R}^{Nm}$, $E_2 \triangleq \left(e_{2,1}^T, e_{2,2}^T, \ldots, e_{2,N}^T\right)^T \in \mathbb{R}^{Nm}$, $\Lambda_1 \triangleq \operatorname{diag}\left(\alpha_{1,1}, \alpha_{1,2}, \ldots, \alpha_{1,N}\right) \otimes I_m \in \mathbb{R}^{Nm\times Nm}$, and I_m is an $m \times m$ identity matrix. Another auxiliary error signal, $R \in \mathbb{R}^{Nm}$, is defined as

$$R \triangleq L_B^{-1}\left(\dot{E}_2 + \Lambda_2 E_2\right), \tag{10}$$

where $\Lambda_2 \triangleq \operatorname{diag}\left(\alpha_{2,1}, \alpha_{2,2}, \ldots, \alpha_{2,N}\right) \otimes I_m \in \mathbb{R}^{Nm\times Nm}$ and $\alpha_{2,i} \in \mathbb{R}$ is a constant positive gain.

IV. Controller Development

The open-loop tracking error system is developed by multiplying (10) by M and utilizing (3), (4) and (8)-(10) to obtain

$$MR = -\tau + d + S_1 + S_2, \tag{11}$$

where the auxiliary functions $S_1 \in \mathbb{R}^{Nm}$ and $S_2 \in \mathbb{R}^{Nm}$ are defined as

$$\begin{aligned} S_1 \;\triangleq\;& M(Q)\, M_\varnothing^{-1}\tau_\varnothing - M(Q_0)\, M_\varnothing^{-1}\tau_\varnothing \\ & -M(Q)\, f_0\left(Q_0, \dot{Q}_0\right) + M(Q_0)\, f_0\left(Q_0, \dot{Q}_0\right) \\ & +f\left(Q, \dot{Q}\right) - f\left(Q_0, \dot{Q}_0\right) \\ & +M(Q)\, L_B^{-1}\left(-\Lambda_1^2 E_1 + \left(\Lambda_1 + \Lambda_2\right) E_2\right) \\ & -M(Q_0)\, L_B^{-1}\left(-\Lambda_1^2 E_1 + \left(\Lambda_1 + \Lambda_2\right) E_2\right) \\ & +M(Q_0)\, L_B^{-1}\left(-\Lambda_1^2 E_1 + \left(\Lambda_1 + \Lambda_2\right) E_2\right), \end{aligned}$$

$$S_2 \triangleq M(Q_0)\, M_\varnothing^{-1}\tau_\varnothing - M(Q_0)\, f_0\left(Q_0, \dot{Q}_0\right) + f\left(Q_0, \dot{Q}_0\right),$$

where the functional dependency of M is given for clarity, and the auxiliary functions $f_0 : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \to \mathbb{R}^{Nm}$ and $f : \mathbb{R}^{Nm} \times \mathbb{R}^{Nm} \to \mathbb{R}^{Nm}$ are defined as

$$f_0\left(Q_0, \dot{Q}_0\right) \triangleq M_\varnothing^{-1}\left(C_\varnothing \dot{Q}_0 + F_\varnothing + G_\varnothing\right), \tag{12}$$

$$f\left(Q, \dot{Q}\right) \triangleq C\dot{Q} + F + G. \tag{13}$$

The RISE-based (c.f. [21], [22]) control input is designed as

$$\tau_i \triangleq \left(k_{s,i} + 1\right) e_{2,i} + \nu_i,\; i = 1, \ldots, N, \tag{14}$$

where $\nu_i \in \mathbb{R}^m$ is the generalized solution to the differential equation

$$\begin{aligned} \dot{\nu}_i \;=\;& \left(k_{s,i} + 1\right)\alpha_{2,i} e_{2,i} \\ & + \sum_{j\in\bar{\mathcal{N}}_i} a_{ij}\left(\chi_i \operatorname{sgn}\left(e_{2,i}\right) - \chi_j \operatorname{sgn}\left(e_{2,j}\right)\right) \\ & + b_i \chi_i \operatorname{sgn}\left(e_{2,i}\right), \quad \nu_i(0) = \nu_{i0}, \end{aligned} \tag{15}$$

where $\nu_{i0} \in \mathbb{R}^m$ is an initial condition, $k_{s,i}, \chi_i \in \mathbb{R}$ are constant positive gains and $\operatorname{sgn}(\cdot)$ is defined $\forall \xi = \begin{bmatrix} \xi_1 & \xi_2 & \ldots & \xi_l \end{bmatrix}^T \in \mathbb{R}^l$ as $\operatorname{sgn}(\xi) \triangleq \begin{bmatrix} \operatorname{sgn}(\xi_1) & \operatorname{sgn}(\xi_2) & \ldots & \operatorname{sgn}(\xi_l) \end{bmatrix}^T$. Note that the controller in (14) is decentralized: only local communication is needed to compute the control authority dictated by the proposed control law.

The following development exploits the fact that the controller in (14) has the time derivative

$$\begin{aligned} \dot{\tau}_i \;=\;& \left(k_{s,i} + 1\right)\left(\dot{e}_{2,i} + \alpha_{2,i} e_{2,i}\right) \\ & + \sum_{j\in\bar{\mathcal{N}}_i} a_{ij}\left(\chi_i \operatorname{sgn}\left(e_{2,i}\right) - \chi_j \operatorname{sgn}\left(e_{2,j}\right)\right) \\ & + b_i \chi_i \operatorname{sgn}\left(e_{2,i}\right). \end{aligned} \tag{16}$$

To facilitate the stability analysis, the time derivative of (11) is expressed as

$$\begin{aligned} M\dot{R} \;=\;& -\frac{1}{2}\dot{M}R + \tilde{N} + L_B N_d - L_B E_2 \\ & -\left(K_s + I_{Nm}\right)\left(\dot{E}_2 + \Lambda_2 E_2\right) \\ & -L_B \beta \operatorname{sgn}\left(E_2\right), \end{aligned} \tag{17}$$

where (16) is expressed in block form as $\dot{\tau} = \left(K_s + I_{Nm}\right)\left(\dot{E}_2 + \Lambda_2 E_2\right) + L_B \beta \operatorname{sgn}\left(E_2\right)$, with $K_s \triangleq \operatorname{diag}\left(k_{s,1}, k_{s,2}, \ldots, k_{s,N}\right) \otimes I_m$ and $\beta \triangleq \operatorname{diag}\left(\chi_1, \chi_2, \ldots, \chi_N\right) \otimes I_m$. In (17), the unmeasurable/uncertain auxiliary terms $\tilde{N} \in \mathbb{R}^{Nm}$ and $N_d \in \mathbb{R}^{Nm}$ are defined as

$$\tilde{N} \triangleq -\frac{1}{2}\dot{M}R + \dot{S}_1 + L_B E_2, \tag{18}$$

$$N_d \triangleq L_B^{-1}\dot{d} + L_B^{-1}\dot{S}_2. \tag{19}$$

The auxiliary terms in (18) and (19) are segregated such that after utilizing (8)-(10), Properties 1-2, Assumptions 1-2, and the Mean Value Theorem, the following upper bounds are satisfied [18]

$$\left\|\tilde{N}\right\| \le \rho\left(\|Z\|\right)\|Z\|, \tag{20}$$

$$\left|N_{d_i}\right| \le \zeta_{a_i},\;\; i = 1, 2, \ldots, Nm, \tag{21}$$

$$\left|\dot{N}_{d_i}\right| \le \zeta_{b_i},\;\; i = 1, 2, \ldots, Nm, \tag{22}$$

where $\rho:\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}$ is a positive nondecreasing function [23, Appendix A], N_{d_i} and $\dot{N}_{d_i}$ denote the i^{th} element of N_d and $\dot{N}_d$, respectively, the elements of $\zeta_a\in\mathbb{R}^{Nm}$ and $\zeta_b\in\mathbb{R}^{Nm}$ are upper bounds on the corresponding elements in N_d and $\dot{N}_d$, respectively, and $Z\in\mathbb{R}^{3Nm}$ is the composite vector

$$Z\triangleq\begin{bmatrix} E_1^T & E_2^T & R^T \end{bmatrix}^T. \tag{23}$$

Note that this bounding process takes advantage of the relations $Q_0-Q=L_B^{-1}E_1$, $\dot{E}_1=E_2-\Lambda_1E_1$, and $\dot{E}_2=L_BR-\Lambda_2E_2$. Additionally, the term $-\frac{1}{2}\dot{M}(Q)R$ in (18) is upper bounded as in (20) by using the relation $Q=Q_0-L_B^{-1}E_1$.

For clarity in the following analysis, let $\zeta_a=\begin{bmatrix}\varsigma_{a1}^T & \varsigma_{a2}^T & \ldots & \varsigma_{aN}^T\end{bmatrix}^T$, where $\varsigma_{ai}\in\mathbb{R}^m$, $i=1,2,\ldots,N$. Similarly, let $\zeta_b=\begin{bmatrix}\varsigma_{b1}^T & \varsigma_{b2}^T & \ldots & \varsigma_{bN}^T\end{bmatrix}^T$, where $\varsigma_{bi}\in\mathbb{R}^m$, $i=1,2,\ldots,N$. Furthermore, define the auxiliary bounding constant as

$$\psi\triangleq\min\left\{\lambda_{\min}(\Lambda_1)-\frac{1}{2},\lambda_{\min}(\Lambda_2)-\frac{1}{2},\lambda_{\min}(L_B)\right\},$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue.

V. Stability Analysis

Theorem 1. *The controller given in (14)-(15) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is semi-globally regulated in the sense that*

$$\|q_0-q_i\|\rightarrow 0\ \ as\ \ t\rightarrow\infty\ (i=1,2,\ldots,N)$$

(and thus $\|q_i-q_j\|\rightarrow 0\ \forall i,j\in\bar{\mathcal{V}}$, $i\neq j$), provided that $k_{s,i}$ introduced in (14) is selected sufficiently large based on the initial conditions of the states (see the subsequent stability analysis), and the parameters $\alpha_{1,i},\alpha_{2,i},\chi_i$ $(i=1,2,\ldots,N)$ are selected according to the sufficient conditions

$$\alpha_{1,i}>\frac{1}{2},\quad \alpha_{2,i}>\frac{1}{2}, \tag{24}$$

$$\chi_i>\|\varsigma_{ai}\|_\infty+\frac{1}{\alpha_{2,i}}\|\varsigma_{bi}\|_\infty, \tag{25}$$

where χ_i was introduced in (15).

Proof: (Sketch) Let $\mathcal{D}\triangleq\left\{y\in\mathbb{R}^{3Nm+1}|\ \rho(\|y\|)<2\sqrt{\psi\,\lambda_{\min}(K_sL_B)}\right\}$ be an open and connected set containing the origin $y=0$, where $y\in\mathbb{R}^{3Nm+1}$ is defined as

$$y\triangleq\begin{bmatrix} Z^T & \sqrt{P}\end{bmatrix}^T. \tag{26}$$

In (26), the auxiliary function $P\in\mathbb{R}$ is the generalized solution to the differential equation

$$\dot{P}=-\left(\dot{E}_2+\Lambda_2E_2\right)^T(N_d-\beta\,\mathrm{sgn}(E_2)) \tag{27}$$

$$P(t_0)=\sum_{j=1}^{Nm}\beta_{j,j}\left|E_{2_j}(t_0)\right|-E_2^T(t_0)\,N_d(t_0),$$

where $\beta_{j,j}$ denotes the j^{th} diagonal element of β and E_{2_j} denotes the j^{th} element of the vector E_2. Provided the sufficient conditions in (25) are satisfied, then $P\geq 0$; the proof is omitted for brevity.

Let $V_L:\mathcal{D}\times\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}$ be a continuously differentiable function defined as

$$V_L\triangleq\frac{1}{2}E_1^TE_1+\frac{1}{2}E_2^TE_2+\frac{1}{2}R^TMR+P. \tag{28}$$

The expression in (28) is positive definite and satisfies the inequalities

$$\lambda_1\|y\|^2\leq V_L(y,t)\leq\lambda_2\|y\|^2, \tag{29}$$

where $\lambda_1\triangleq\frac{1}{2}\min\{1,m_{min}\}$, $m_{min}\triangleq\min_{j\in\bar{\mathcal{V}}}\left(\underline{m}_j\right)$, and $\lambda_2\triangleq\max\left\{1,\frac{1}{2}\sum_{j=1}^N\overline{m}_j(q_j)\right\}$, provided the sufficient conditions in (24) and (25) are satisfied.

Consider the set $S_{\mathcal{D}}\subset\mathcal{D}$ defined as

$$\mathcal{S}_{\mathcal{D}}\triangleq\left\{y\in\mathcal{D}|\rho\left(\sqrt{\frac{\lambda_2}{\lambda_1}}\|y\|\right)<2\sqrt{\psi\,\lambda_{\min}(K_sL_B)}\right\}. \tag{30}$$

Under Filippov's framework, a Filippov solution y can be established for the closed-loop system $\dot{y}=h(y,t)$ if $y(t_0)\in S_{\mathcal{D}}$, where $h:\mathbb{R}^{3Nm+1}\times\mathbb{R}_{\geq 0}\rightarrow\mathbb{R}^{3Nm+1}$ denotes the right-hand side of the closed-loop error signals. It can be shown that the time derivative of (28) exists almost everywhere (a.e.), i.e., for almost all $t\in[t_0,t_f]$. After using the upper bound in (20), the Raleigh-Ritz theorem, and Young's inequality, the time derivative of (28) can be upper-bounded as

$$\begin{aligned}\dot{V}_L\ &\overset{a.e.}{\leq}\ -\psi\|Z\|^2-\lambda_{\min}(K_sL_B)\|R\|^2\\ &\quad+\rho(\|Z\|)\|R\|\|Z\|.\end{aligned} \tag{31}$$

Completing the squares for terms in (31) yields

$$\dot{V}_L\overset{a.e.}{\leq}-\left(\psi-\frac{\rho^2(\|Z\|)}{4\,\lambda_{\min}(K_sL_B)}\right)\|Z\|^2. \tag{32}$$

The expression in (32) can be further upper-bounded by

$$\dot{V}_L\overset{a.e.}{\leq}-c\|Z\|^2 \tag{33}$$

for all $y\in\mathcal{D}$, for some positive constant $c\in\mathbb{R}$.

The inequalities in (29) and (33) can be used to show that $V\in\mathcal{L}_\infty$. Thus, $E_1,E_2,R\in\mathcal{L}_\infty$. The closed-loop error system can be used to conclude that the remaining signals are bounded. From (33), [24, Corollary 1] can be invoked to show that $c\|Z\|^2\rightarrow 0$ as $t\rightarrow\infty\ \forall y(t_0)\in\mathcal{S}_{\mathcal{D}}$. Based on the definition of Z in (23), $\|E_1\|\rightarrow 0$ as $t\rightarrow\infty\ \forall y(t_0)\in\mathcal{S}_{\mathcal{D}}$. Noting the definition of E_1 in (8) and the fact that $\left(\left(\bar{\mathcal{L}}+B\right)\otimes I_m\right)$ is full rank, it is clear that $\|Q_0-Q\|\rightarrow 0$ as $t\rightarrow\infty$ if and only if $\|E_1\|\rightarrow 0$ as $t\rightarrow\infty$. Thus, $\|q_0-q_i\|\rightarrow 0$ as $t\rightarrow\infty\,\forall i\in\bar{\mathcal{V}}$. It logically follows that $\|q_i-q_j\|\rightarrow 0$ as $t\rightarrow\infty\,\forall i,j\in\bar{\mathcal{V}}$, $i\neq j$. ■

Note that the region of attraction in (30) can be made arbitrarily large to include any initial conditions by adjusting the control gains $k_{s,i}$ (i.e., a semi-global result).

The distributed controller shown in Section IV is decentralized in the sense that only local feedback is necessary to compute the controller value. However, note that because the constant gain $k_{s,i}$ must be selected based on sufficient conditions involving L_B, which contains information regarding the configuration of the entire network, this gain is chosen in a centralized manner before the control law is implemented.

VI. Simulation

To demonstrate the performance of the developed controller, simulation results are presented for the synchronization of four follower agents to a leader's state trajectory. Each network follower is modeled as a two-link robotic manipulator (a typical example of an Euler-Lagrange system) with the form

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} p_1 + 2p_3c_2 & p_2 + p_3c_2 \\ p_2 + p_3c_2 & p_2 \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -p_3s_2\dot{q}_2 & -p_3s_2(\dot{q}_1 + \dot{q}_2) \\ p_3s_2\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_{d1} & 0 \\ 0 & f_{d2} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + d_\tau,$$

where q_1, q_2 denote joint angles, $c_2 \triangleq \cos(q_2)$, $s_2 \triangleq \sin(q_2)$, τ_1 and τ_2 represent torque control inputs, and d_τ represents the added disturbances. The constant unknown parameters $p_1, p_2, p_3, f_{d1}, f_{d2} \in \mathbb{R}$ differ for each manipulator. The virtual leader is defined by the trajectory $q_0 = \begin{bmatrix} 2\sin(2t) \\ \cos(3t) \end{bmatrix}$, where the first and second elements are the desired trajectories for the first and second joint angles, respectively. The time-varying disturbance term has the form $d_\tau = \begin{bmatrix} a\sin(bt) \\ c\sin(dt) \end{bmatrix}$, where the constants $a, b, c, d \in \mathbb{R}$ differ for each manipulator. The control policy in (14) is implemented as $\tau_i \triangleq (k_{s,i} + 1)(e_{2,i} - e_{2,i}(0)) + \nu_i$ with $\nu_{i0} = 0$, where the term $e_{2,i}(0)$ is included so that $\tau_i(0) = 0$ and has no impact on the stability result (i.e., (16) remains the same). Additionally, in an effort to improve the gain tuning procedure, the gains $k_{s,i}$ and χ_i are implemented as diagonal matrices such that $k_{s,i} = \text{diag}(k_{s1,i}, k_{s2,i})$ and $\chi_i = \text{diag}(\chi_{1,i}, \chi_{2,i})$. The network topology is shown in Fig. 1, the simulation parameters are shown in Table I, and the network performance is provided in Fig. 2 and 3. The network shown in Fig. 1 has an undirected and connected follower network. Fig. 2 demonstrates that asymptotic synchronization of the follower agents and tracking of the leader trajectory are achieved, despite the exogenous disturbances. Fig. 3 shows the low control effort of the individual controllers.

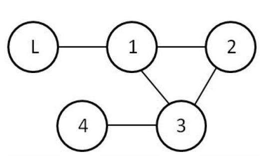

Figure 1. Network Topology

	Robot 1	Robot 2	Robot 3	Robot 4
p_1	3.7	3.5	3.2	3.0
p_2	0.22	0.20	0.18	0.17
p_3	0.19	0.25	0.23	0.21
f_{d1}	5.3	5.1	5.2	5.4
f_{d2}	1.1	1.3	1.0	1.2
a	2.0	4.0	3.0	5.0
b	1.0	2.0	3.0	5.0
c	1.0	3.0	4.0	2.0
d	4.0	3.0	1.0	2.0
k_a	1.0	1.0	1.0	1.0
k_b	4.0	4.0	4.0	4.0
k_{s1}	210	240	210	240
k_{s2}	21	24	21	24
χ_1	75	75	75	75
χ_2	0.075	0.075	0.075	0.075
α_1	1.5	1.8	1.5	1.5
α_2	1.5	1.5	1.5	1.5

Table I
Simulation Parameters

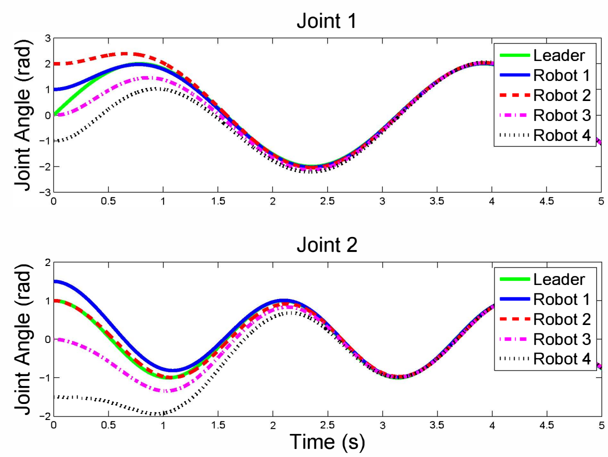

Figure 2. Joint Angles

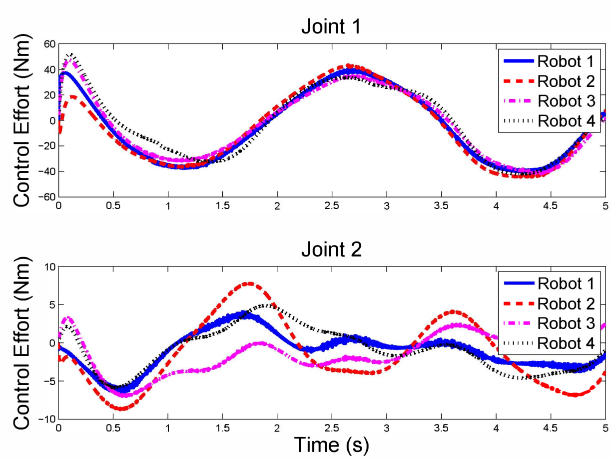

Figure 3. Control Effort

VII. Conclusion

A decentralized RISE-based controller was developed which ensures semi-global asymptotic tracking and synchronization of networked followers' states towards a leader's time-varying state using a continuous control input, despite model uncertainty and exogenous disturbances, where the leader and follower agents have uncertain and heterogeneous Euler-Lagrange dynamics. The graph of the networked follower agents is assumed to be connected and at least one follower agent receives information from the leader. Simulation results are provided for the proposed decentralized controller.

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